Números y polinomios de Bernoulli

$$S_k(n) = \sum_{1\le i\le n} i^k = 1^k + 2^k + \cdots + n^k$$

$$S_k(n) = \frac{1}{k+1}\left((n+1)^{k+1} - 1 - \sum_{0\le i\le k-1} \binom{k+1}{i} S_i(n) \right)$$

$$\sum_{0\le i\le k} \binom{k+1}{i} B_i = k+1 \qquad \longleftrightarrow \qquad \frac{t\,e^t}{e^t-1} = \sum_{k\ge 0} B_k \frac{t^k}{k!}$$

k:	0	1	2	3	4	5	6	7	8	9	10
B_k:	1	$\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{30}$	0	$\frac{1}{42}$	0	$-\frac{1}{30}$	0	$\frac{5}{66}$
k:		11	12	13	14	15	16	17	18	19	20
B_k:		0	$-\frac{691}{2730}$	0	$\frac{7}{6}$	0	$-\frac{3617}{510}$	0	$\frac{43867}{798}$	0	$-\frac{174611}{330}$

$$\frac{t\,e^{tx}}{e^t-1} = \sum_{k\ge 0} B_k(x) \frac{t^k}{k!}$$

$$B_k = S'_k(0) \qquad\qquad B_k(1) = B_k,\ B_k(0) = B_k \text{ para } k \ne 1$$

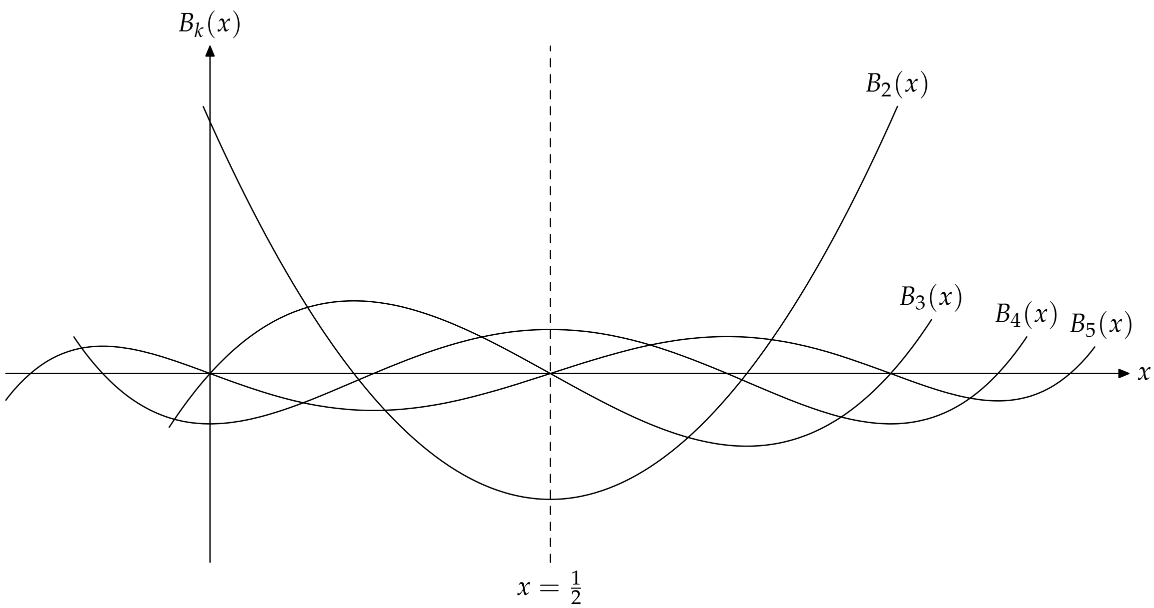
$$S'_k(x) = k\,S_{k-1}(x) + B_k \qquad\qquad B'_k(x) = k\,B_{k-1}(x)$$

$$S_k(x) = \frac{1}{k+1} \sum_{0\le i\le k} \binom{k+1}{i} B_i\, x^{k+1-i} \qquad\qquad B_k(x) = \sum_{0\le i\le k} (-1)^i \binom{k}{i} B_i\, x^{k-i}$$

$$B_k(x) = S'_k(x) \text{ para } k \ne 1$$

$$S_0(x) = x$$
$$S_1(x) = \frac{1}{2}x^2 + \frac{1}{2}x$$
$$S_2(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}x$$
$$S_3(x) = \frac{1}{4}x^4 + \frac{1}{2}x^3 + \frac{1}{4}x^2$$
$$S_4(x) = \frac{1}{5}x^5 + \frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{1}{30}x$$
$$S_5(x) = \frac{1}{6}x^6 + \frac{1}{2}x^5 + \frac{5}{12}x^4 - \frac{1}{12}x^2$$
$$S_6(x) = \frac{1}{7}x^7 + \frac{1}{2}x^6 + \frac{1}{2}x^5 - \frac{1}{6}x^3 + \frac{1}{42}x$$
$$S_7(x) = \frac{1}{8}x^8 + \frac{1}{2}x^7 + \frac{7}{12}x^6 - \frac{7}{24}x^4 + \frac{1}{12}x^2$$
$$S_8(x) = \frac{1}{9}x^9 + \frac{1}{2}x^8 + \frac{2}{3}x^7 - \frac{7}{15}x^5 + \frac{2}{9}x^3 - \frac{1}{30}x$$
$$S_9(x) = \frac{1}{10}x^{10} + \frac{1}{2}x^9 + \frac{3}{4}x^8 - \frac{7}{10}x^6 + \frac{1}{2}x^4 - \frac{3}{20}x^2$$
$$S_{10}(x) = \frac{1}{11}x^{11} + \frac{1}{2}x^{10} + \frac{5}{6}x^9 - x^7 + x^5 - \frac{1}{2}x^3 + \frac{5}{66}x$$

$$B_0(x) = 1$$
$$B_1(x) = x - \frac{1}{2}$$
$$B_2(x) = x^2 - x + \frac{1}{6}$$
$$B_3(x) = x^3 - \frac{3}{2}x^2 + \frac{1}{2}x,$$
$$B_4(x) = x^4 - 2x^3 + x^2 - \frac{1}{30}$$
$$B_5(x) = x^5 - \frac{5}{2}x^4 + \frac{5}{3}x^3 - \frac{1}{6}x$$
$$B_6(x) = x^6 - 3x^5 + \frac{5}{2}x^4 - \frac{1}{2}x^2 + \frac{1}{42},$$
$$B_7(x) = x^7 - \frac{7}{2}x^6 + \frac{7}{2}x^5 - \frac{7}{6}x^3 + \frac{1}{6}x$$
$$B_8(x) = x^8 - 4x^7 + \frac{14}{3}x^6 - \frac{7}{3}x^4 + \frac{2}{3}x^2 - \frac{1}{30},$$
$$B_9(x) = x^9 - \frac{9}{2}x^8 + 6x^7 - \frac{21}{5}x^5 + 2x^3 - \frac{3}{10}x$$
$$B_{10}(x) = x^{10} - 5x^9 + \frac{15}{2}x^8 - 7x^6 + 5x^4 - \frac{3}{2}x^2 + \frac{5}{66}$$



$$B_k(1-x) = (-1)^k B_k(x)$$
$$\int_0^1 B_k(x)\,dx = 0$$